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# Quantum corrections to higher dimensional theories

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**Abstract**
This is a non-technical summary of the subtleties of quantum corrections on extra-dimensional theories: should one first renormalize and then mode expand, or first expand in four-dimensional modes and then renormalize?

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In the past few years there has been an increasing interest in field theories defined in spacetimes of dimension greater than 4. Such models, seen as low-energy effective theories of a more fundamental consistent theory like superstrings, provide a new variety of very interesting mechanisms in order to solve long standing problems of the standard model.

Interesting possibilities are the idea of the Higgs particle originated from extra-dimensional components of gauge fields [11], often called gauge-Higgs unification, and alternative mechanisms for symmetry breaking [10, 14]. The best known proposals of this kind are probably large extra dimensions [3] and warped scenarios [13]. These are only the original references, although the literature on the matter is very extensive.

A common problem in higher dimensional models is the necessity to explain why extra dimensions are hidden, in the sense that the spacetime we experiment is effectively four dimensional. Traditionally, extra dimensions are supposed to be compact and with a characteristic size extremely small[3] so that we would need energies unattainables in present colliders in order to directly detect them. Compactness of the extra dimensions allows us to expand fields propagating in the whole spacetime in harmonics and perform integrals over the extra coordinates. In that way we find a four-dimensional theory, but with an infinite number of fields corresponding to modes of the expansion: the so-called Kaluza–Klein modes.

We can then distinguish two viewpoints, the higher dimensional and the four dimensional with the tower. They are of course completely equivalent at the classical level. The question we are trying to answer is if this last statement remains true, and if so under what conditions, when one consider quantum corrections on both points of view. A negative result will be important because, as far as we know, the calculations in the literature are almost always performed in four dimensions taking into account the tower (cf however [8]). If the correct

[3] It is possible to avoid this requirement by using warped geometries with localized gravity [13].

way to understand higher dimensional field theories is to compute radiative corrections directly in the complete spacetime (as we will try to argue), then a great number of results for the models considered before should be examined.

If we are interested in quantum effects it is sufficient to work to one-loop order. To this order, the effective action is given in terms of a functional determinant:

$$\Gamma \sim \log\det\Delta, \tag{1}$$

where $\Delta$ is the operator representing the quadratic part of the action. In many interesting theories, for instance the standard model, this last quantity is divergent. Extraction of the divergent part of (1) in a consistent way is the process of renormalization, in this case to one loop. There are several ways of identifying the divergent part of (1), for example, diagrammatically in the sense of 't Hooft's algorithm [15] generalized to the appropriate dimension[4]. A more effective approach, specially on curved backgrounds, is the heat kernel [5]. Both kind of computations make use of the background field method explained also in [5].

Concerning higher dimensional theories, it is then obvious that quantum equivalence requires the matching of the divergences on both points of view. The aim of this work is to explore whether this matching is possible or not.

It is important to say that in the particular case of a scalar interacting only through the universal coupling to an external gravitational field, after solving some subtleties, it is possible to perform a clever resummation of the modes in a way that divergences coincide, although it is true that we find counterterms that we should not expect in a purely four-dimensional computation, as shown by Duff and Toms in [6].

A crucial point in the argument is that the operators considered can be split into the form

$$\Delta = \Delta_1 + \Delta_2, \tag{2}$$

where $\Delta_1$ acts trivially on the extra-dimensional coordinates and $\Delta_2$ acts trivially on the usual four-dimensional ones[5]. Therefore, the result is not valid when the splitting does not take place, as happens on a warped background[6] as well as for a general interacting theory.

We will focus our attention on a simple interacting theory, in particular quantum electrodynamics defined on a six-dimensional manifold $\mathbb{R}^4 \times S^1 \times S^1$. The Euclidean version of the action is

$$S = \int \mathrm{d}^6x \left(\frac{1}{4}F_{MN}^2 + \bar{\psi}(\not{D} + m)\psi\right). \tag{3}$$

An informed reader may note that this action is non-renormalizable, since the gauge coupling has mass dimension $[e_6] = -1$. However, up to one loop this fact is not important in the sense that we can still identify and study all the divergences. Before performing any calculation let us think a moment what should we expect to find when one considers the one-loop correction.

As is well known, the counterterms of the theory will be the most general six-dimensional operators compatible with the symmetries of the system, in this case a $U(1)$ gauge symmetry and Lorentz invariance. Thus the dimensionality of the coupling allows us to write terms such as

$$e^2 D_M F^{MN} D^R F_{RN}, \qquad e^2 D_R F_{MN} D^R F^{MN}. \tag{4}$$

Despite they were not present in the original Lagrangian, the radiative generation of these operators is unavoidable: the power of the coupling shows that is a one-loop effect, they are of the right dimension and have the correct invariance. The appearance or this terms was

[4] This algorithm is designed to give the poles in dimensional regularization in four dimensions, but it can be easily generalized to an arbitrary *even* dimension. If one uses a proper time cutoff instead of dimensional regularization it can also be applied to odd dimensions.

[5] This requires a factorizable metric, so the gravitational background is not arbitrary.

[6] A hint in that direction is given in [7].

discussed in [9, 12]. Another important point is that the symmetry forbids a mass term for the gauge field, so the bosonic zero modes remain massless at one loop.

The explicit six-dimensional computation performed in [1] agrees with these intuitions.

In order to perform a four-dimensional computation with the whole KK tower, let us expand the fields in modes. Compactification of (3) gives the four-dimensional gauge fixed action:

$$\begin{aligned} S = \int \mathrm{d}^4x \sum_{n_5,n_6} \Bigg( & \bar{\psi}^1_n \partial\!\!\!/ \psi^1_n + \bar{\psi}^2_n \partial\!\!\!/ \psi^2_n + \bar{\psi}^1_n \left( \mathrm{i}\frac{n_5}{R_5} + \frac{n_6}{R_6} \right) \psi^2_n - \bar{\psi}^2_n \left( \mathrm{i}\frac{n_5}{R_5} - \frac{n_6}{R_6} \right) \psi^1_n \\ & + m \left( \bar{\psi}^1_n \psi^1_n - \bar{\psi}^2_n \psi^2_n \right) - \frac{1}{2} (A^n_\mu)^* \left( \Box - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A^\mu_n \\ & - \frac{1}{2} (A^n_5)^* \left( \Box - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A^n_5 - \frac{1}{2} (A^n_6)^* \left( \Box - \frac{n_5^2}{R_5^2} - \frac{n_6^2}{R_6^2} \right) A^n_6 \\ & - e \sum_m \left( \bar{\psi}^1_m A\!\!\!/_{m-n} \psi^1_n + \bar{\psi}^2_m A\!\!\!/_{m-n} \psi^2_n + \bar{\psi}^1_m A^{m-n}_5 \psi^2_n \right. \\ & \left. - \bar{\psi}^2_m A^{m-n}_5 \psi^1_n - \mathrm{i}\bar{\psi}^1_m A^{m-n}_6 \psi^2_n - \mathrm{i}\bar{\psi}^2_m A^{m-n}_6 \psi^1_n \right) \Bigg). \end{aligned} \tag{5}$$

One has to double the number of fermions because in $d$ dimensions they have $2^{[d/2]}$ components (eight in six dimensions, four in four dimensions). Also the extra components of the gauge field $A^n_5$ and $A^n_6$ appear as four-dimensional scalars[7]. It is important to note that the spacetime symmetry is spontaneously broken to

$$O(6) \longrightarrow O(4) \times O(2) \times O(2). \tag{6}$$

While the extra-dimensional gauge symmetry traduces into the infinite set of four-dimensional symmetries:

$$\begin{aligned} \delta A^n_\mu &= \mathrm{i}\partial_\mu \Lambda_n \\ \delta A^n_5 &= -\frac{n_5}{R_5}\Lambda_n \\ \delta A^n_6 &= -\frac{n_6}{R_6}\Lambda_n. \end{aligned} \tag{7}$$

Please note that the scalar zero modes are singlets under a gauge transformation. Finally, the coupling is now dimensionless, as it is defined by

$$e \equiv \frac{e_6}{\sqrt{R_5 R_6}} \equiv e_6 M. \tag{8}$$

Let us repeat the exercise done with the previous action and ask ourselves what kind of corrections one would expect. First of all, the coupling is dimensionless so we cannot use it to reduce the dimension of higher order operators. Therefore, terms like the ones in (4) are forbidden, at least in perturbation theory.

Next, since the scalar zero mode is singlet there are no symmetries to protect its mass against radiative corrections, as it happens with the standard model Higgs. Then we expect a mass term for it (in fact there is no reason not to expect operators of higher power, i.e. cubic or quartic interactions).

Another important point is that the gauge zero mode $A^0_\mu$, which plays the role of the usual photon, couples diagonally to an infinite tower of fermions, with the same strength as in four-dimensional QED and to every fermion. The only difference between the fermions of the

[7] They are identified with the Higgs in gauge-Higgs unification.

tower is their masses, which are labelled by a pair of integers. Now, the pole in the vacuum polarization function does not depend on the mass of the fermion running in the loop. We should have then the same contribution to the $\beta$-function as in QED for every fermion. Since the number of fermions is infinite, one has to sum the same quantity an infinite number of times. This gives rise to an additional divergence coming from the sum. One can think that this is the expected effect of an infinite number of fields interacting all to each other.

Again all these expectations are confirmed with standard computations and the explicit result can be found in [1]. Of course, it seems impossible to reconcile both points of view. A natural question is to what extent this is the consequence of the non-renormalizability of the model. Unfortunately, studies along these lines but with a renormalizable theory (in particular four-dimensional QED) show that the inequivalence has nothing to do with renormalizability.

In fact, the case of $QED_4$ on the four-dimensional manifold $\mathbb{R}^2 \times S^1 \times S^1$ provides the most transparent example of this kind of effects, so it is worth studying it.

The counterterm calculated in the whole spacetime is the usual one of QED, which yields the well-known $\beta$-function:

$$\beta = \frac{e^3}{12\pi^2}, \tag{9}$$

or in terms of the two-dimensional coupling $\bar{e} = eM$:

$$\bar{e}^2 = \frac{\bar{e}_0^2}{1 - \frac{\bar{e}_0^2}{6\pi^2 M^2} \log \frac{\mu}{\mu_0}}. \tag{10}$$

On the other hand, symmetry forbids again a mass term form the gauge boson. Moreover, in four dimensions even if we include explicitly a mass term for the gauge boson in the bare Lagrangian its mass does not receive radiative corrections and remains unrenormalized [4].

From a two-dimensional perspective the situation is radically different. The superficial degree of divergence of a diagram is now

$$D = 2 - \tfrac{1}{2} E_f - V, \tag{11}$$

where $V$ is the number of vertices and $E_f$ is the number of fermionic external lines. This means that any diagram with fermions in external lines can be primitively divergent. Thus, there are no counterterms for the fermionic sector, a fact that is impossible to justify thinking in four dimensions. The primitively divergent diagrams involve only bosons as external states.

Moreover, the vacuum polarization function is known to be finite in two dimensions (remember the Schwinger model). This means that the only divergent correction, apart from a tadpole, is the two-point function of the two-dimensional scalars $A_3^n$ and $A_4^n$. The zero mode was massless at tree level, but now since it is a gauge singlet it gets mass through radiative corrections. This was impossible in four dimensions as we have said. Also there is no running of the coupling at all, in clear contradiction with (10), although possible deviations from $e^2 \approx e_0^2$ can be seen only in energies exponential in the compactification mass:

$$\frac{\mu}{\mu_0} \gg \mathrm{e}^{\frac{6\pi^2 M^2}{\bar{e}_0^2}}. \tag{12}$$

The explicit counterterm is given in [1], but its properties are basically the ones explained here.

The conclusion is that there is some sort of quantum inequivalence between Kaluza–Klein models when one considers loop corrections in the whole spacetime or in the dimensionally reduced theory. Therefore, one has to take care when computing in extra-dimensional field theories, at least when dealing with radiative corrections. In that case, since one considers effects at energies much higher than the compactification scale, the compact dimensions should be seen in the same way as the usual four and the spacetime should be higher dimensional. The natural way of computing is then in the whole manifold performing next the mode expansion to get four-dimensional quantities. A more detailed argumentation of this way of thinking can be found in [2].

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